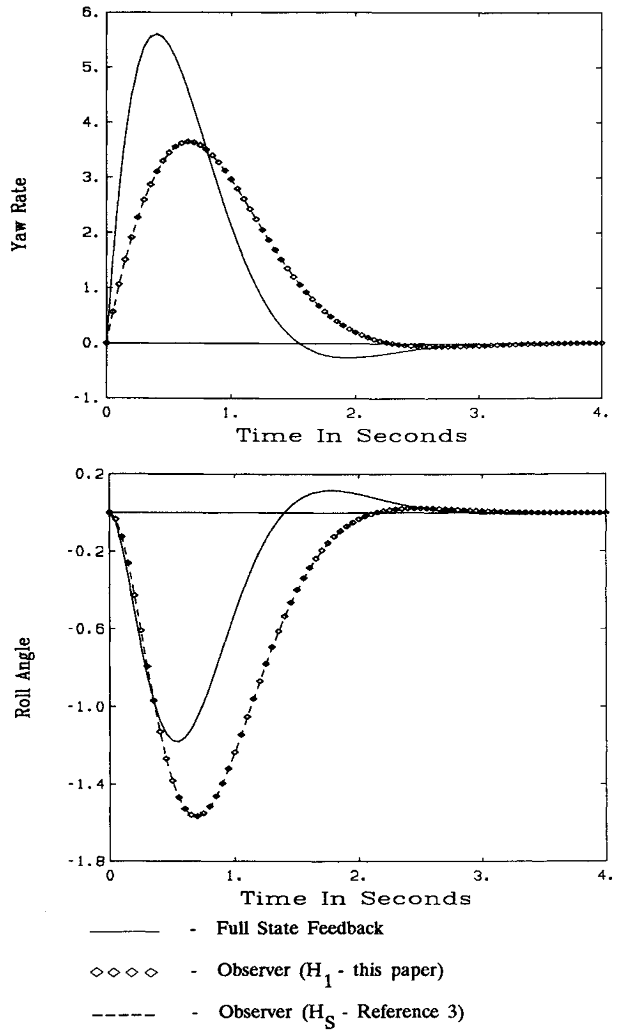

Fig. 1 Actual aircraft states for each control law.

Table 1 Comparison of the gain matrices H_1 and H_S

Property	H_1 (this paper)	H_S (Ref. 3)
$J_1^{1/2}$	12.30	13.44
J_2	10.95	11.23
$J_3^{1/2}$	6.61	6.74
c_1	5.49	5.62
c_2	1.00	3.07
c_3	1.01	3.47
c_4	5.49	5.39
$\|c\|_2$	7.89	9.06

illustration of the fact that reducing a bound on the norm of the gain matrix does not imply that this norm will be reduced.

For a gust-induced initial sideslip, let the initial condition of the state vector be $x(0) = (0, 0, 0, 5)^T[= -e(0)]$. Time responses of some of the actual states of the aircraft are given in Fig. 1 for the full state control law and each of the systems formed by using the observer gain matrices H_1 and H_S. Note that the closed-loop systems including the observers exhibit an almost identical time response with gain matrices of similar norm. However, the design using the method introduced in this paper (H_1) possesses reduced overall eigenvalue sensitivity compared to the old design (H_S).

A design with reduced eigenvalue sensitivity or with a lower norm gain matrix, etc., may be obtained by allowing the closed-loop eigenvalues to be selected in conjugate pairs from regions of the complex plane. Alternatively, one could place more emphasis on certain design objectives by altering the weighting factors α, β, and γ.

Conclusion

A multiple objective optimization technique using analytically derived gradients has been described. This technique allows for the design of an observer that has low eigenvalue sensitivity to parameter variation or uncertainty in the system matrices and is achieved using a small (in norm) gain matrix that is of reduced sensitivity to measurement error. Supplementary to this, the impact of a known structure (or direction) for an initial condition mismatch is reduced by assigning the left eigenstructure of the observer.

The new method provides improved results over former methods because of the use of analytically derived derivatives of the multiple objective cost function. The earlier methods used numerically derived gradients that may sometimes provide a non-optimal solution. In addition, provision is made so that the design freedom may be increased by allowing closed-loop eigenvalues to be chosen from regions of the complex plane.

One is able to place more (or less) emphasis on the single objectives contained in the overall cost function by the adjustment of certain weighting factors that have been introduced.

References

[1]Andry, A. N., Chung, J. C., and Shapiro, E. Y., "Modalized Observers," *IEEE Transactions on Automatic Control,* Vol. 29, No. 7, 1984, pp. 669–671.

[2]Andry, A. N., Shapiro, E. Y., and Chung, J. C., "Eigenstructure Assignment for Linear Systems," *IEEE Transactions on Aerospace and Electronic Systems,* Vol. 19, No 5, 1983, pp. 711–729.

[3]Sobel, K. M., and Banda, S. S., "Design of a Modalized Observer with Eigenvalue Sensitivity Reduction," *Journal of Guidance, Control, and Dynamics,* Vol. 12, No. 5, 1989, pp. 762–763.

[4]Gill, P. E., Murrary, W., and Wright, M. H., *Practical Optimization,* Academic Press, New York, 1981.

[5]Franklin, G. F., Powell, J. D., and Emami-Naeini, A., *Feedback Control of Dynamic Systems,* Addison-Wesley, Reading, MA, 1986.

[6]Wilkinson, J. H., *The Algebraic Eigenvalue Problem,* Oxford University Press, U.K., 1965.

[7]Golub, G. H., and Van Loan, C. F., *Matrix Computations,* North Oxford Academic, 1983.

[8]Roppenecker, G., "Minimum Norm Output Feedback Design Under Specified Eigenvalue Areas," *Systems & Control Letters,* Vol. 3, No. 2, 1983, pp. 101–103.

[9]Kautsky, J., Nichols, N. K., and Van Dooren, P., "Robust Pole Assignment in Linear State Feedback," *International Journal of Control,* Vol. 41, No. 5, 1985, pp. 1129–1155.

Pulse Response Method for Vibration Reduction in Periodic Dynamic Systems

Carl R. Knospe* and John Kenneth Haviland†
University of Virginia, Charlottesville, Virginia 22936

Introduction

SEVERAL periodic dynamic systems possess unwanted vibration, including the helicopter, which has received much attention recently. Periodic systems cannot be described in terms of classical transfer functions or time-invariant state transition matrices. Thus, conventional control theory, such

Received April 27, 1990; revision received Oct. 12, 1990; accepted for publication Dec. 31, 1990.

*Assistant Professor, Department of Mechanical and Aerospace Engineering.

†Professor of Aerospace Engineering, Department of Mechanical and Aerospace Engineering.

as classical and state space, cannot be applied to these systems. Periodic dynamics can best be described in terms of Floquet theory.[1] However, there are no control methods at present based on this theory. Periodic systems can be expressed in terms of a response matrix together with an uncontrolled vibration vector in either the discrete frequency or the discrete-time domains. If these quantities do not vary with time, their characteristics can be measured, and a fixed gain system can be designed. However, if these quantities vary, as with the helicopter, an adaptive system is needed.

This Note describes the application of the pulse response method to the active alleviation of vibrations in periodic systems. A great deal of research has focused on active alleviation of helicopter vibrations in induced canceling vibrations in the fuselage through small modifications in the helicopter blades' angle of attack. These control concepts have focused on the reduction of blade passing frequency vibrations through higher harmonic controllers.[2-4]

In the present paper, adaptive vibration reduction is achieved through a cautious controller in the discrete-time domain using a pulse response formulation, as opposed to a frequency domain formulation. Since the acceleration measurements are not prefiltered to blade passing frequency before the regulator estimates the parameters of the linear model relating vibration and control, the resulting algorithm reduces vibrations at all harmonics (within Nyquist limitations) simultaneously.

Pulse Response Method

For any linear system, the response at discrete times to a pulse control sequence can be represented by the matrix equation

$$\begin{bmatrix} \vdots \\ x(t_{j-1}) \\ x(t_j) \\ x(t_{j+1}) \\ \vdots \end{bmatrix} = \begin{bmatrix} \cdots & 0 & 0 & 0 \\ \cdots & w(t_{j-1,j-1}) & 0 & 0 \\ \cdots & w(t_{j,j-1}) & w(t_{j,j}) & 0 \\ \cdots & w(t_{j+1,j-1}) & w(t_{j+1,j}) & w(t_{j+1,j+1}) \\ \cdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$\times \begin{bmatrix} \vdots \\ u(t_{j-1}) \\ u(t_j) \\ u(t_{j+1}) \\ \vdots \end{bmatrix} + \begin{bmatrix} \vdots \\ x_0(t_{j-i}) \\ x_0(t_j) \\ x_0(t_{j+1}) \\ \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ y(t_{j-1}) \\ y(t_j) \\ y(t_{j+1}) \\ \vdots \end{bmatrix}$$

$$= \begin{bmatrix} \vdots \\ x(t_{j-1}) \\ x(t_j) \\ x(t_{j+1}) \\ \vdots \end{bmatrix} + \begin{bmatrix} \vdots \\ v(t_{j-1}) \\ v(t_j) \\ v(t_{j+1}) \\ \vdots \end{bmatrix}$$

where x, x_0, y, u, and v are the vibration, uncontrolled vibration, vibration measurement, control, and measurement noise, respectively. Note that if the disturbance responses $x_0(t_j)$ and the control pulse response $w(t_j, t_k)$(i.e., the weighting sequence) are predictable, then an open-loop control can be found to minimize a performance index of x and u. If n measurements are taken per cycle, where a cycle is the period of the system dynamics, and if the control induced vibrations damp out in p cycles, then the matrix equations become

$$\boldsymbol{x}_i = \sum_{l=0}^{p} \boldsymbol{T}_{i,i-l}\boldsymbol{u}_{i-l} + \boldsymbol{x}_{0_i}, \qquad y_i = x_i + v_i \tag{1}$$

where the pulse response $T_{i,i-l}$ relates cycle $i-l$ input to cycle i output and

$$\boldsymbol{x}_i = \begin{bmatrix} x(t_{in+1}) \\ \vdots \\ x(t_{in+n}) \end{bmatrix}, \quad \boldsymbol{u}_i = \begin{bmatrix} u(t_{in+1}) \\ \vdots \\ u(t_{in+n}) \end{bmatrix}$$

$$\boldsymbol{x}_{0_i} = \begin{bmatrix} x_0(t_{in+1}) \\ \vdots \\ x_0(t_{in+n}) \end{bmatrix}, \quad y_i = \begin{bmatrix} y(t_{in+1}) \\ \vdots \\ y(t_{in+n}) \end{bmatrix}, \quad v_i = \begin{bmatrix} v(t_{in+1}) \\ \vdots \\ v(t_{in+n}) \end{bmatrix}$$

Assuming near periodicity of the plant dynamics and uncontrolled response, the control vector will be nearly constant between cycles and the sum can be taken over the T_{ij} matrices in Eq. (1) yielding

$$\boldsymbol{x}_i = \boldsymbol{T}_i\boldsymbol{u}_i + \boldsymbol{x}_{0_i}, \qquad y_i = x_i + v_i \tag{2}$$

or

$$\boldsymbol{x} = \boldsymbol{\Theta}\underline{\boldsymbol{u}}$$

$$\boldsymbol{\Theta} = [\boldsymbol{x}_0 | \boldsymbol{T}], \qquad \underline{\boldsymbol{u}} = [1 | \boldsymbol{u}^T]^T \tag{3}$$

where the cycle count subscript i has been suppressed. Note that $\boldsymbol{T}$ is an $n \times n$ matrix whereas $\boldsymbol{y}$, $\boldsymbol{u}$, and $\boldsymbol{x}$ are n vectors.

Identification Algorithm

For many periodic dynamic systems such as helicopters, the dynamics and disturbances undergo slow changes. For the purpose of deriving an identification algorithm, these variations are modeled by the random walk in the pulse response matrix and uncontrolled vibration vector

$$\boldsymbol{\Theta}_{i+1} = \boldsymbol{\Theta}_i + w_{\theta,i} \tag{4}$$

where $w_{\theta,i}$ is a matrix of random variations in the base vibration vector $\boldsymbol{x}_0$ and pulse response matrix $\boldsymbol{T}$. The Kalman filter used here parallels that used by Molusis et al.[3] and Bryson and Ho[5] and therefore will not be rederived. The algorithm used,

$$\bar{\boldsymbol{P}}_{i+1} = \bar{\boldsymbol{P}}_i - \bar{\boldsymbol{P}}_i\underline{\boldsymbol{u}}_i\boldsymbol{K}_i^T + \boldsymbol{W}$$

$$\bar{\boldsymbol{\Theta}}_{i+1} = \hat{\boldsymbol{\Theta}}_i = \bar{\boldsymbol{\Theta}}_i + (y_i - \bar{\boldsymbol{\Theta}}_i\underline{\boldsymbol{u}}_i)\boldsymbol{K}_i^T \tag{5}$$

relates the successive a priori values of $\bar{\boldsymbol{P}}$ and $\bar{\boldsymbol{\Theta}}$, the covariance, and estimate matrices. Here, $\boldsymbol{K}$ is the Kalman gain vector

$$\boldsymbol{K}_i = \bar{\boldsymbol{P}}_i\underline{\boldsymbol{u}}_i / (\underline{\boldsymbol{u}}_i^T\bar{\boldsymbol{P}}_i\underline{\boldsymbol{u}}_i + V)$$

whereas

$$V = E\{v^T v\} = n\sigma_V^2$$

$$W = E\{(\boldsymbol{\Theta}_{i+1} - \boldsymbol{\Theta}_i)^T(\boldsymbol{\Theta}_{i+1} - \boldsymbol{\Theta}_i)\} = \text{diag}\{n\sigma_X^2, n\sigma_T^2, \ldots, n\sigma_T^2\}$$

Control Algorithm

The role of the control algorithm is to calculate the control vector $\underline{\boldsymbol{u}}$ for each cycle using the plant and uncontrolled vibration estimates. In deriving the control from a minimization of a quadratic performance function, two complications result. First, because y is not available until the end of a cycle, $\bar{\boldsymbol{P}}$ and $\bar{\boldsymbol{\Theta}}$ cannot be calculated in time for use in the next cycle. It is therefore necessary to adjust the control algorithm to allow for this delay. Second, the formulation of the system dynamics in the discrete-time domain (as opposed to the narrowband frequency domain of higher harmonic control) allows the control to initiate rigid-body motion. For the helicopter problem, a constraint must be included in the control law to prevent inducement of such modes so that the vibration control does not oppose the pilot's control. The control input is therefore calculated subject to the constraint

$$u^T\Psi = 0 \tag{6}$$

where Ψ is a matrix whose columns represent the undesirable control modes. The columns of the matrix Ψ are the control vectors that produce rigid-body motion. Introducing Λ, a vector of Lagrange multipliers, and minimizing the quadratic performance function

$$J = E\{x^Tx + u^Tu(\alpha) + u^T\Psi\Lambda + \Lambda^T\Psi^Tu\} \tag{7}$$

yields the control law

$$u = \{Z^{-1}\Psi[\Psi^TZ^{-1}\Psi]^{-1}\Psi^T - I\}Z^{-1}X \tag{8}$$

where

$$X_{i+1} = \bar{T}_i^T\bar{x}_{0,i} + \bar{P}_{21,i}$$

$$Z_{i+1} = \bar{T}_i^T\bar{T}_i + \bar{P}_{22,i} + I\alpha + W_{22}$$

The $n \times 1$ vector $\bar{P}_{21}$ and the $n \times n$ matrix $\bar{P}_{22}$ are partitions of $\bar{P}$:

$$\bar{P} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix}$$

The covariances expressed by the $\bar{P}$ matrix result from the definition of the performance function J in terms of expected values; this type of controller is referred to as being "cautious." Note that control for cycle $i + 1$ is computed during cycle i. Since the output y_i is not available until the end of cycle i, new a posteriori estimates of the parameters are unavailable for the computation of the control. The control derived therefore is optimal with respect to an information state that is one cycle old.

Because of the rigid-body mode constraints on the control, any collection of control vectors does not span the control n space. No information along the constrained vectors of the parameter space is obtained, presenting an estimation problem. This is manifested as the error covariance matrix increasing without bound and bears a resemblance to the persistency of excitation problem of stochastic control. The remedy used here, referred to as probing, is the introduction of noise into the control vector and is the same as advocated by Wittenmark[6] to prevent "turn off" in cautious controllers. While this violates the control constraint, it results in little rigid-body motion. Another solution to this problem is renormalizing the covariance matrix with its trace to prevent the instability.

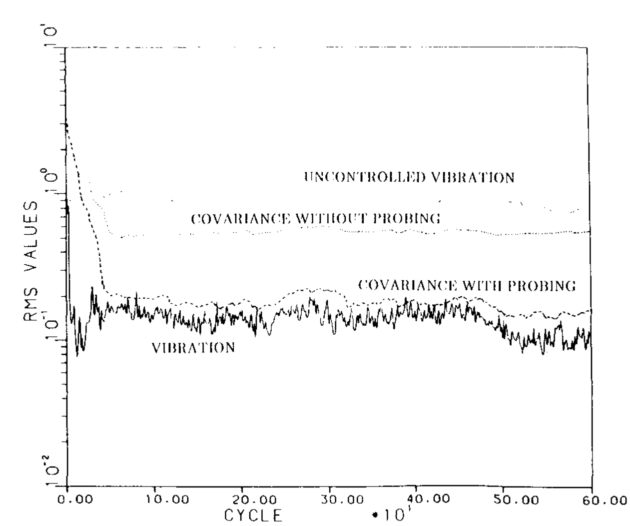

Fig. 1 Controlled and uncontrolled root mean square vibration with probing, and covariance trace with and without probing.

Application to Helicopter Vibration Reduction

Assuming steady flight with well-balanced blades, the dynamics of a helicopter and the controlled vibrations to which it is subjected are approximately periodic and are repeated every blade passage. Thus, the cycle for this problem is equal to one quarter of a revolution for a four-bladed helicopter. With four accelerometers, full individual blade control of a four-bladed rotor would be possible, yielding vibration control on all axes. To simplify for the example problem, only vertical vibration control will be considered here. Thus, collective pitch is driven by feedbacks from a vertically mounted accelerometer, and each blade receives the same control input. The control algorithm derived previously is employed with the constraint vector

$$\Psi = [1 \quad 1 \quad 1 \quad 1 \ldots 1]^T$$

which would nominally preclude a change in lift and thus prevent helicopter climb or descent. The actual value of this constraint vector, however, is not important to the example presented; the presence of a constraint on the control, however, affects the algorithm's function dramatically.

For the purpose of computer simulation, the model plant's pulse response at each azimuthal location was varied stochastically in a random walk. With an actual plant, the tracking and control of the algorithm would not be significantly different.

As has been reported previously,[7] preliminary numerical simulation results displayed regulator instability. This was manifested in the slow continues growth of the elements of the covariance matrix. At first, this behavior was thought to be the result of poor tuning (i.e., choice of filter process noise terms). The instability problem was, in fact, more fundamental than poor tuning; it was due to an optimal control that was not sufficiently rich (i.e., did not span the parameter space) to stabilize the system. To correct the problem, a small pseudorandom signal, referred to as probing, was inserted into the control for excitation. This was found to bound the covariances and stabilize the system. Simulation results in Fig. 1 show this effect. Shown are the root-mean-square values over one cycle of the uncontrolled and controlled vibration vectors with probing and the trace of the covariance matrix for the algorithm with and without probing. Note that the covariance trace without the probing term is significantly greater than that with probing; further results not displayed show continuing growth when probing is not used, indicating true instability. The changes in levels of vibration reduction over the time history are due to changes in the plant pulse response matrix and not to any instability in the regulator algorithm. Note that the algorithm typically yields approximately 15 dB of vibration reduction when probing is included. Without probing, vibration levels were slightly lower (not shown). Once sufficient probing has been inserted to bound the covariance, additional probing causes unnecessary degradation of performance. Preliminary research shows that covariance bounding also results in a stable regulator and good vibration reduction with none of the adverse effects of probing.

Conclusions

The pulse response formulation of periodic dynamic systems permits the adaptive vibration control problem to be solved in the discrete-time domain. Kalman filtering may be employed for identification, but specific issues of availability of the estimates and covariance instability must be resolved in the control algorithm design. Probing can prevent covariance instability due to insufficient richness of the control at the expense of a small loss in attenuation performance.

Acknowledgment

This work was supported in part by the Army Research Office.

References

[1]Lukes, D. L., *Differential Equations: Classical to Controlled*, Academic, New York, 1982, Chap. 8.

[2]Kretz, M., Aubrun, J. N., and Larch, M., "Wind Tunnel Tests of the Dorand DH 2011 Jet Flap Rotor," Vol. 1, NASA-CR-114693, June 1973.

[3]Molusis, J. A., Hammond, C. E., and Cline, J. H., "A Unified Approach to the Optimal Design of Adaptive and Gain Scheduled Controllers to Achieve Minimum Helicopter Rotor Vibration," 37th Annual Forum of the American Helicopter Society, New Orleans, LA, May 1981.

[4]Johnson, W., "Self Tuning Regulators for Multicyclic Control of Helicopter Vibration," NASA TP 1996, March 1982.

[5]Bryson, A. E., and Ho, Y. C., *Applied Optimal Control: Optimization, Estimation, and Control*, Hemisphere, New York, 1975, Chap. 12.

[6]Wittenmark, B., "Stochastic Adaptive Control Methods: A Survey," *International Journal of Control*, Vol. 21, No. 5, 1975, pp. 705–730.

[7]Haviland, J. K., and Knospe, C., "A Wide-Band, Time-Domain Adaptive Control for Helicopter Vibrations," Second Technical Workshop on Dynamics and Aeroelastic Stability Modeling of Rotorcraft Systems, Boca Raton, FL, Nov. 1987.

Robust Flight Control System Design with Multiple Model Approach

Yoshikazu Miyazawa*

National Aerospace Laboratory, Chofu, Tokyo, Japan

Introduction

CONTROL system design for uncertain dynamical systems and systems with changing parameters has been one of the major research subjects in linear control theory. Recently, new theoretical results have been introduced using the frequency response, such as the H_∞ approach, and these results have been attempted to be extensively applied to flight control problems. The multiple model approach discussed here has been developed with the same objective (e.g., Ref. 1). The plant dynamics is not uniquely given, but it is described using multiple candidates of dynamical systems or multiple models.

The concept of multiple models has long been used in flight control system design. Changes of parameters, such as dynamic pressure, Mach number, weight and balance, and configuration, have a significant influence on the dynamic properties of aircraft, and consideration of every necessary point in the flight envelope is important for the design of a flight control system. Although such design attempts have often been carried out in a trial-and-error manner based on empirical knowledge, an extension of control theory with the multiple model approach has been proposed for more efficient design. The multiple model approach has been combined with linear quadratic regulator (LQR),[2,3] linear quadratic Gaussian (LQG),[3-5] and pole assignment.[1]

For flight control system design, not only prescribed changes of parameters but also uncertain dynamics should be considered. Uncertainty in the high frequency range, which derives from various sources such as flexible modes, nonlinearity of the actuator system, uncertain time lags due to digital signal processing, and the effects of filters, is frequently pointed out for careful consideration. In practice, designers often find that high gain feedback control introduces a high crossover frequency, where phase information is quite uncertain, and this makes the closed-loop system unstable.[6] This is one of the major motivations of recent robust control research. In the robust control problems, how to present uncertainty of the dynamical system is important. In the present Note, an uncertain delay model is proposed to present uncertainty in the high frequency range. The multiple model approach with an uncertain delay model can introduce the maximum regulator performance while suppressing the frequency bandwidths of feedback control. By considering different times of delay, bandwidths can be adjusted for different points of the closed-loop system.

The present multiple model approach was successfully applied to an active flutter control problem.[7] The approach is applied to a flight control problem emphasizing regulator performance and bandwidth for the multi-input control system. A prefixed control structure, such as proportional output feedback with a fixed gain, can provide a practical control law, and it is useful to avoid acute dependency on the selected points for multiple models.[8] Numerical results show practical feasibility of the approach for flight control system design.

Multiple Model Approach

The LQR methodology is extended to the multiple model problem while posing a constraint of proportional output feedback. The following is a brief review of the approach. The plant dynamics is given by the following multiple models:

$$\frac{dx_i}{dt} = A_i x_i(t) + B_i u_i(t), \qquad y_i(t) = C_i x_i(t); \qquad i = 1, M \tag{1}$$

where $x_i \in R^{n_i}$ $u_i \in R^m$, $y_i \in R^r$ are the state variable, control variable, and output variable, respectively. The subscript i denotes the ith model, and the system matrices (A_i, B_i, C_i) are constant matrices of adequate size. The initial condition of state variables is considered random, and its characteristic is defined as

$$E[x_i(0)] = 0, \qquad E[x_i(0)x_i^T(0)] = W_i; \qquad i = 1, M \tag{2}$$

where $E[\ ]$ denotes the average, and $W_i(\geq 0)$ is appropriately given. The control law is defined as follows:

$$u_i(t) = Ky_i(t) \tag{3}$$

The feedback gain K is common to all models. The output y_i may be a variable that is directly measured or a variable that is accurately reconstructed with a filter.

When the closed-loop systems, $A_i + B_iKC_i$, are asymptotically stable, the performance index is defined as a weighted summation of performance indices for each model, i.e.,

$$J = \sum_{i=1}^{M} p_i J_i, \quad J_i = E\left[\int_0^\infty x_i(t)^T Q_i x_i(t) + u_i(t)^T R u_i(t)\, dt\right] \tag{4}$$

where $p_i(>0, \sum p_i = 1)$ is the probability of the ith model, or it can be used as an adjustable design parameter. Weighting matrices $Q_i(\geq)$ and $R(>0)$ are appropriately given as the standard LQR problem. The performance index can be rewritten as

$$J = \sum_{i=1}^{M} p_i \operatorname{tr}[(Q_i + C_i^T K^T R K C_i)X_i] \tag{5}$$

$$(A_i + B_iKC_i)X_i + X_i(A_i + B_iKC_i)^T + W_i = 0$$

$$X_i = E\left[\int_0^\infty x_i(t)x_i(t)^T\, dt\right] \tag{6}$$

The optimal gain that minimizes the performance index satisfies the following necessary conditions or the optimality conditions:

$$\sum_{i=1}^{M} p_i R K_i C_i X_i C_i^T + p_i B_i^T Y_i X_i C_i^T = 0 \tag{7}$$

Received June 28, 1990; presented, in part, as Paper 90-3411 at the AIAA Guidance, Navigation, and Control Conference, Portland, OR, Aug. 20-22, 1990; revision received Dec. 13, 1990; accepted for publication Dec. 26, 1990.

*Senior Researcher, Flight Research Division, 6-13-1 Osawa, Mitaka, Tokyo 181 Japan. Member AIAA.